**Computational Logic**

**Content:**

**I. Numeration systems, numbers representations**

**II. Classical logics: propositional and predicate logics**

### III. Boolean algebras, Boolean functions and logic circuits

1. **NUMERATION SYSTEMS**

**Positional systems:** Hindu-Arabic numeration systems

**Examples:**

* binary system: base = 2, digits : 0,1 , **ex**: **11001(2)**
* octal system: base = 8, digits: 0,1,2,3,4,5,6,7, **ex: 2047(8)**
* decimal system: base =10, digits: 0,1,2,3,4,5,6,7,8,9, **ex**: **2343**
* hexadecimal system: base =16, digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

where: A (16) =10, B (16) =11, C (16) =12, D (16) =13, E (16) =14, F (16) =15,  **ex: 2AF(16)**

**A numeric value (7) has different representations:**

**7 (decimal), VII (Roman system), 111(binary), şapte (Romanian language), seven (English)**

**Remarks:**

The position of a digit in a representation implies an association with a “**positional value**”.

The **numeric value** is the sum of the positional values of all the digits from the representation.

1. The calculation of the **numeric value (decimal value)** of a representation is the **conversion from an arbitrary base into decimal.**

**Example 1:**

2 3 4 3 = 3\*100  + 4\*101  + 3\*102 + 2\*103 = 3 + 40 + 300 + 2000

│ │ │ └─-- 3\*100 = 3 (positional value)

│ │ └──--- 4\*101 = 40 (positional value)

│ └───---- 3\*102 = 300 (positional value)

└────----- 2\*103 = 2000 (positional value)

**Example 2:**

1 3 7 3(8) = ? (10)

**Example 3:**

1 1 0 1 (2) = ? (10)

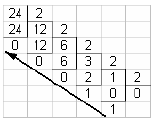
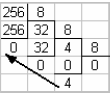
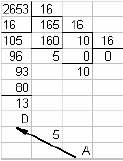
**Example 4:**

AC9 (16) = ?(10)

1. **Conversion from decimal into an arbitrary base – successive divisions**

**Examples (F.Boian):**

24 = 11000(2) 256 = 400(8) 2653 = A5D (16)

**Example 5:**

**256 = ?(5)**

**Operations with integers in different numeration bases**

1. **Addition in base b: s(b) = x(b) + y(b)**

**x, y – terms, s - sum**

**x = (xn ….x1 x0)(b) , y = (yn ….y1 y0)(b) s = (sn+1 sn ….s1 s0)(b)**

**Example 6: (b=10)**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |
| **x** | **=** |  | **9** | **8** | **4** | **3** | **+** |
| **y** | **=** |  |  | **5** | **6** | **7** |  |
| **s** | **=** |  |  |  |  |  |  |

**Algorithm:**

**c0=0; //c0, c1,… ,cn+1 Ԑ {0,1}are the carries used in addition**

**for i=0,n**

**x’i (10) = xi (b) ; y’i (10) = yi (b) ; //convert the digits from base b in base 10**

**s10 = x’i (10)+y’i (10) +ci (10) ; //addition in base 10**

**s’i (10) = s10 mod b ; ci+1 = s10 div b;**

**si (b) = s’i (10); //convert the decimal value in base b**

**end\_for**

**si+1 = ci+1;**

**Example 7:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Positions (i)** | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| **Operation** | **Base (b)** | **Carries (c)** |  |  |  |  |  |  |  |  |  |  |
| **+** | **7** | **x** |  |  |  |  |  |  |  |  |  |  |
|  |  | **y** |  |  |  |  |  |  |  |  |  |  |
|  |  | **Sum (s)** |  |  |  |  |  |  |  |  |  |  |

**Calculations:**

**Example 8:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Positions (i)** | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| **Operation** | **Base (b)** | **Carries (c)** |  |  |  |  |  |  |  |  |  |  |
| **+** | **2** | **x** |  |  |  |  |  |  |  |  |  |  |
|  |  | **y** |  |  |  |  |  |  |  |  |  |  |
|  |  | **Sum (s)** |  |  |  |  |  |  |  |  |  |  |

**Calculations:**

**Example 9 :**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Positions (i)** | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| **Operation** | **Base (b)** | **Carries (c)** |  |  |  |  |  |  |  |  |  |  |
| **+** | **16** | **x** |  |  |  |  |  |  |  |  |  |  |
|  |  | **y** |  |  |  |  |  |  |  |  |  |  |
|  |  | **Sum (s)** |  |  |  |  |  |  |  |  |  |  |

**Calculations:**

1. **Multiplication by one digit in base b: p(b) = x(b) \* f(b) , x, f –factors, p-product**

**x = (xn ….x1 x0)(b) , f(b) - one digit**

**p = (pn+1 pn ….p1 p0)(b)**

**Example 10 (b=10):**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
| **x** | **=** |  | **2** | **3** | **4** | **\*** |
| **f** | **=** |  |  |  | **7** |  |
| **p** | **=** |  |  |  |  |  |

**Algorithm:**

**c0=0; //c0, c1,… ,cn+1 are the carries used in multiplication**

**for i=0,n**

**x’i (10) = xi (b) ; f ’(10) = f (b) ; //convert the digits from base b in base 10**

**p10 = x’i (10)\*f ’(10) +ci (10) ; //operations in base 10**

**p’i (10) = p10 mod b ; ci+1 = p10 div b;**

**pi (b) = p’i (10); //convert the decimal value in base b**

**end\_for**

**pi+1 = ci+1;**

**Example 11:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Positions (i)** | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| **Operation** | **Base (b)** | **Carries (c)** |  |  |  |  |  |  |  |  |  |  |
| **\*** | **8** | **x** |  |  |  |  |  |  |  |  |  |  |
|  |  | **f** |  |  |  |  |  |  |  |  |  |  |
|  |  | **Product (p)** |  |  |  |  |  |  |  |  |  |  |

**Calculations:**

**Example 12:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Positions (i)** | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| **Operation** | **Base (b)** | **Carries (c)** |  |  |  |  |  |  |  |  |  |  |
| **\*** | **6** | **x** |  |  |  |  |  |  |  |  |  |  |
|  |  | **f** |  |  |  |  |  |  |  |  |  |  |
|  |  | **Product (p)** |  |  |  |  |  |  |  |  |  |  |

**Calculations:**

**Example 13:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Positions (i)** | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| **Operation** | **Base (b)** | **Carries (c)** |  |  |  |  |  |  |  |  |  |  |
| **\*** | **16** | **x** |  |  |  |  |  |  |  |  |  |  |
|  |  | **f** |  |  |  |  |  |  |  |  |  |  |
|  |  | **Product (p)** |  |  |  |  |  |  |  |  |  |  |

**Calculations:**

1. **Division by one digit in base b: x(b) : f(b) = q (b) and r(b) ,**

**x- dividend, f – divisor, q – quotient, r - remainder**

**x = (xn ….x1 x0)(b) , f(b) - one digit,q = (qn ….q1 q0)(b) , 0 <=r(b) < f(b)**

**Example 14 (b=10):**

**x(dividend)**

2043 | 5 f(divisor)

/ | q(quotient)

/

/

/

r (remainder)

**Algorithm**

**r’ = 0; // r’ – a decimal value**

**for i = n,0,-1**

**x’i (10) = xi (b) ; //convert the digits from base b in base 10**

**p10 = r’\*b + x’i (10) ; //operations in base 10**

**q’i (10) = p10 div f ; r’= mod f;**

**qi (b) = q’i (10) //convert the decimal value in base b**

**end\_for**

**r(b) =r’(10) //convert the decimal value in base b**

|  |  |
| --- | --- |
| **Example 15:**  **x(dividend) f(divisor)**  02543(8) |5(8)  / | (8) – q(quotient)  25  /  14  /  23  /  4(8) – r (remainder) | **Calculations:**  **02(8)=0\*8+2= 2, 2 div 5=0, 2 mod 5=2**  **25 (8)=2\*8+5=21, 21 div 5=4, 21 mod 5=1**  **14(8) = 1\*8 + 4 = 12, 12 div 5 = 2, 12 mod 5 = 2**  **23(8)=16+3=19, 19 div 5= 3, 19 mod 5= 4** |

|  |  |
| --- | --- |
| **Example 16:**  **x(dividend) f(divisor)**  02043(6) | 5(6)  / | (6) – q(quotient)    /    /    /  (6) – r (remainder) | **Calculations:** |

|  |  |
| --- | --- |
| **Example 17:**  **x(dividend) f(divisor)**  0B2CF(8) | A(16)  / | (16) – q(quotient)    /    /    /  (16) – r (remainder) | **Calculations:** |

1. **Subtraction in base b: d(b) = x(b) - y(b) ,**

**x - minuend, y - subtrahend, d - difference**

**x = (xn ….x1 x0)(b) , y = (yn ….y1 y0)(b) d = (dn ….d1 d0)(b)**

**Example 18 (b=10) :**

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| **x** | **=** | **8** | **1** | **0** | **0** | **4** | **-** |  | **8** | **1** | **0** | **0** | **4** | **-** |
| **y** | **=** |  | **4** | **5** | **6** | **7** |  |  |  | **4** | **5** | **6** | **7** |  |
| **d** | **=** |  |  |  |  |  |  |  |  |  |  |  |  |  |

**Algorithm**

**c0=0; //c0, c1,… ,cn+1 Ԑ {0,1}are the borrows used in subtraction**

**for i=0,n**

**x’i (10) = xi (b) ; y’i (10) = yi (b); //convert the digits from base b in base 10**

**d10 = x’i(10) – ci (10) –y’i (10); //subtraction in base 10**

**ci+1 = 0;**

**if (d10 < 0 ) then { d10 = d10+b ; ci+1 = 1;}**

**di (b)= d10(10) //convert the decimal value in base b**

**end\_for**

**Example 19:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Positions (i)** | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| **Operation** | **Base (b)** | **Borrows (c)** |  |  |  |  |  |  |  |  |  |  |
| **-** | **8** | **x** |  |  |  |  |  |  |  |  |  |  |
|  |  | **y** |  |  |  |  |  |  |  |  |  |  |
|  |  | **Difference(d)** |  |  |  |  |  |  |  |  |  |  |

**Calculations:**

**Example 20:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Positions (i)** | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| **Operation** | **Base (b)** | **Borrows (c)** |  |  |  |  |  |  |  |  |  |  |
| **-** | **2** | **x** |  |  |  |  |  |  |  |  |  |  |
|  |  | **y** |  |  |  |  |  |  |  |  |  |  |
|  |  | **Difference(d)** |  |  |  |  |  |  |  |  |  |  |

**Calculations:**

**Example 21:**

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | **Positions (i)** | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |
| **Operation** | **Base (b)** | **Borrows (c)** |  |  |  |  |  |  |  |  |  |  |
| **-** | **16** | **x** |  |  |  |  |  |  |  |  |  |  |
|  |  | **y** |  |  |  |  |  |  |  |  |  |  |
|  |  | **Difference(d)** |  |  |  |  |  |  |  |  |  |  |

**Calculations:**